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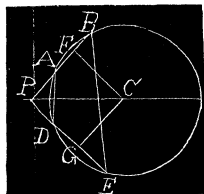
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$$\begin{aligned}
\therefore \Delta &= aR \int_{\theta'}^{\theta''} [\sqrt{1-e^2 \sin^2 \theta} \sin \theta + \sqrt{1-e^2 \cos^2 \theta} \cos \theta] d\theta / \int_{\theta'}^{\theta''} d\theta \\
&= \frac{aR}{4 \sin^{-1}(R/a) - \pi} \left[\sin \theta' \sqrt{1-e^2 \cos^2 \theta'} \right. \\
&\quad - \sin \theta'' \sqrt{1-e^2 \cos^2 \theta''} - \cos \theta' \sqrt{1-e^2 \sin^2 \theta'} \\
&\quad \left. + \cos \theta'' \sqrt{1-e^2 \sin^2 \theta''} \right] \\
&+ \frac{1-e^2}{e} \log \left(\frac{e \sin \theta' + \sqrt{1-e^2 \cos^2 \theta'}}{e \sin \theta'' + \sqrt{1-e^2 \cos^2 \theta''}} \right) \\
&\quad - \frac{1-e^2}{e} \log \left(\frac{e \cos \theta' + \sqrt{1-e^2 \sin^2 \theta'}}{e \cos \theta'' + \sqrt{1-e^2 \sin^2 \theta''}} \right) \Big].
\end{aligned}$$



But $\sin \theta' = \cos \theta'' = R/a$, $\sin \theta'' = \cos \theta' = [\sqrt{a^2 - R^2}]/a$.
 $e^2 \sin^2 \theta' = e^2 \cos^2 \theta'' = 1$, $e^2 \sin^2 \theta'' = e^2 \cos^2 \theta' = [(a^2 - R^2)/R^2] = e^2 - 1$.
 Whence by substitution and reduction we get

$$\Delta = \frac{R^2}{2 \sin^{-1}(R/a) - \frac{1}{2} \pi} \left[\sqrt{1-e^2} + (1-e^2) \log \left(\frac{1 + \sqrt{1-e^2}}{\sqrt{1-e^2}} \right) \right].$$

83. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find the average area of all ellipses whose semi-axis major is a .

I. Solution by J. W. YOUNG, Fellow and Assistant, Ohio State University, Columbus, Ohio.

The area of an ellipse whose major-axis is a , and whose minor-axis is b , is πab . We must find the average of all possible values of this expression as b varies from zero to a .

$$\begin{aligned}
&\frac{\pi a \int_0^a b db}{\int_0^a db} \\
\therefore \text{Average required} &= \frac{\pi a \int_0^a b db}{\int_0^a db} = \frac{1}{2} \pi a^2,
\end{aligned}$$

$= \frac{1}{2}$ the area of the circle whose radius is the major-axis of the ellipse.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

1. Let x = semi-conjugate axis. Then average area

$$\begin{aligned}
&\frac{\int_0^a x dx}{\int_0^a dx} \\
&= \frac{\pi a \int_0^a x dx}{\int_0^a dx} = \frac{1}{2} \pi a^2 = 1.5708 a^2.
\end{aligned}$$

2. Let e =eccentricity. Then $\text{area}=\pi a^2\sqrt{1-e^2}$.

$$\therefore \text{Average area}=\pi a^2 \frac{\int_0^1 \sqrt{1-e^2}}{\int_0^1 de} = \frac{1}{2}\pi^2 a^2 = 2.4674a^2.$$

MISCELLANEOUS.

73. Proposed by CHAS. E. MYERS, Canton, Ohio.

In an ice cream freezer, cream of a homogeneous character and at the uniform temperature of 60° Fahrenheit is put into a cylinder having a closed base, and the whole put into a freezing mixture so as to subject the base and convex surface to a constant temperature of 30° Fahrenheit. Required the temperature at any point within the cream after the expiration of a given time. [From *Higher Mathematics*.]

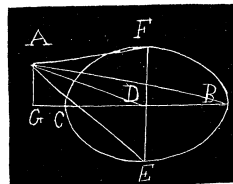
No solution of this problem has been received.

74. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

The longest diameter of a horizontal ellipse is $CB=2a=6$ feet. Its shortest diameter is $EF=2b=4$ feet, their intersection being at D . Find in an indefinite vertical plane passing through CB , a point $A=5$ feet= c from D , the ellipse being seen from A as a circle.

I. Solution by the late B. F. BURLESON, and the PROPOSER.

The eye being at A , and the ellipse being projected as a circle, CB and EF subtend equal angles at A , or $\angle EAF=\angle BAC$. Produce DC to G , A being vertically over G , and put $CG=x$, and $GA=y$, and $\angle ADC=\phi$ =angle of elevation of A .



$$\text{Then } y=\sqrt{c^2-(a+x)^2} \dots\dots (1).$$

$$AB=\sqrt{(2a+x)^2+y^2} \dots\dots (\alpha).$$

$$\sin \angle ACG=\sin \angle ACB=y/\sqrt{x^2+y^2} \dots\dots (\beta), \text{ and } \tan \angle EAD=b/c \dots\dots (\gamma).$$

$$\therefore \sin \angle EAF=\sin \angle BAC=2dc/(b^2+c^2) \dots\dots (\delta).$$

From $\triangle BAC$ we have the proportion, $AB : \sin \angle ACB :: BC : \sin \angle BAC$.

$$\therefore \frac{2bc\sqrt{(2a+x)^2+y^2}}{b^2+c^2} = \frac{2ay}{\sqrt{x^2+y^2}} \dots\dots (2).$$

$$\text{Resolving (1) and (2) we have } x=\frac{c\sqrt{(c^4-a^2b^2)(a^2-b^2)}-a}{a(c^2-b^2)}$$

$$=\frac{5}{63}\sqrt{(2945)-3}=1.30697255 \text{ feet.}$$

$$\therefore y=\frac{bc(c^2-a^2)}{a(c^2-b^2)}=2\frac{3}{4} \text{ feet.}$$